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SELF MAGNETIC INSULATION OF PULSED ION DIODES

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Abstract—Pulsed ion diodes magnetically insulated by their own feed currents can be treated in a manner similar to that used for applied field diodes. The self insulated diode has the advantages of easy construction and automatic confinement of the magnetic field to the accelerating gap. On the other hand, the energy that produces the magnetic field is utilized inefficiently. With models for the ion current density, it is possible to calculate the optimum electrode shapes of ion diodes to produce a spherical focus. In this application it is shown that the pinched electron diode may not be the most advantageous self insulated geometry.

THE RECENT invention of electrode geometries that expedite the efficient production of intense pulsed ion beams has brought about an increased interest in the use of focused ion beams to achieve fusion by inertial confinement (WINTERBERG, 1975; CLAUSER, 1975; SHEARER, 1975). Three geometries that have been successful in producing multikiloampere ion beams are the reflex triode (HUMPHRIES *et al.*, 1974; PRONO *et al.*, 1975; KAPETANAKOS *et al.*, 1975), the magnetically insulated diode (DREIKE *et al.*, 1976; HUMPHRIES *et al.*, 1976) and the pinched electron pinch, in fact, may not be the most advantageous configuration for methods rely on magnetic fields to inhibit electron flow across the accelerating gap to achieve good efficiencies. In this paper a common ground of description for these two types of diodes will be developed, allowing comparison of the relative advantages of each. In these terms it can be seen that the pinched electron diode is a special case of the general class of self magnetically insulated diodes. The electron pinch, in fact, may not be the most advantageous configuration for obtaining an optimum ion focus. A method for determining the optimum shape of focusing ion diodes will be presented.

A number of authors have considered the magnetic insulation of stressed vacuum gaps against electron flow (ROSTOKER, 1972; LOVELACE *et al.*, 1974) and its application to ion acceleration (WINTERBERG, 1968; SUDAN *et al.*, 1973). If there is a uniform electric field in a gap (E_x produced by a voltage V_0 across a gap d) in the presence of a uniform magnetic field, B_z, then conservation of the canonical momentum p_y gives the criterion on the minimum magnetic field needed to prevent an electron from crossing the gap,

$$B^* = \frac{(2eV_0/r_e)^{1/2}}{d}(1+eV_0/2m_ec^2)^{1/2}. \tag{1}$$

Here, r_e is the classical electron radius and the second factor in parenthesis is a relativistic correction. A number of theories have considered the modifications introduced by the inclusion of electron and ion space charge effects (SUDAN *et al.*, 1973; BERGERON, 1976; ANTONSEN *et al.*, 1976). In general, it is found that (a) the criterion for insulation against electron flow is about that given in (1), (b) the ion current density crossing the gap can be somewhat enhanced above the Child–Langmuir limit when $B \gtrsim B^*$, and (c) for experimentally practical values of B ($B > 1.5B^*$) the ion current density is close to the Child-Langmuir limit.

Although the transversely drifting electron space charge does not greatly affect the small scale behavior of the diode, it can be an important consideration in building real systems. There is no magnetically insulated geometry in which the electron drift orbits can be perfectly closed. Even in a magnetron geometry there will be some component of B_θ from the current feeds that can be large when dealing with multikiloampere devices. Discontinuous cathode surfaces may be associated with electron losses. For the simple geometry of the parallel plate gap with uniform magnetic field, an upper limit on such losses can be derived. The geometry is shown in Fig. 1(a). It is obvious that there can be no steady state for

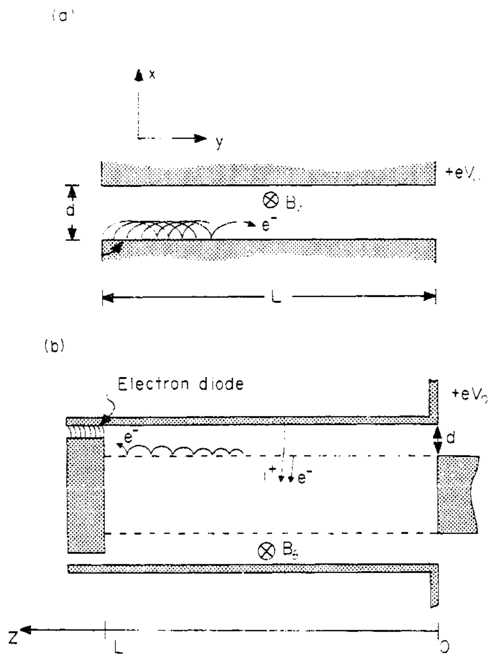

FIG. 1.—(a) Magnetically insulated finite planar gap. Electron emitted from Region A can sustain the equilibrium electron sheath distribution within the gap. (b) Cylindrical self magnetically insulated ion focusing diode.

drifting electrons reaching the end of the plates. The decrease in E_x lowers the drift velocity (approximately cE_x/B_z) and the electron sheath thickness so that the electron flux must be discontinuous. A lower estimate on the ratio of drifting electron current to crossing ion current can be obtained following arguments of BLAUGRUND *et al.* (1975). Since the electric field at both electrodes must be zero (space charge limited flow), the ratio of electron to ion currents is inversely proportional to their crossing times. Taking $t_e = L/(cE_x/B_z)$ and $t_i = (2dm_i/eE_x)^{1/2}$ and approximating E_x as V_0/d gives

$$I_e I_i \cong (B^*/B)(d/L)(m_i/m_e)^{1/2}. \quad (2)$$

In order for an electron in the accelerating gap to return to the cathode, it must arrive with zero kinetic energy and zero v_x which can only occur when the symmetry of the system is maintained. An upper limit on the total electron current is obtained by assuming that all the drifting electrons that arrive at the end of the plates are transported across the gap. The actual loss may be lower. The crossing electrons may be transported by losing energy to other electrons by non-equilibrium processes. These electrons would then have enough kinetic energy to return to the cathode.

In treating ion focusing geometries, it is convenient to divide the problem of ion optics into microscopic and macroscopic regimes. The microscopic problem concerns the minimization of the local beam divergence angle. This has been studied in great detail for steady state ion accelerators (see for instance, COOPER *et al.*, 1974) and is in an initial state of development for intense pulsed accelerators (HUMPHRIES *et al.*, 1976). In the macroscopic problem the main concern is the general shape of the electrodes to aim all the ions at a central point. The complicating factor for magnetically insulated diodes is the deflection suffered by the ions in crossing the accelerating gap. In the absence of this effect, the ideal electrode shape would obviously be a spherical section. For small deflections, the angle can be written

$$\alpha \cong (m_e/m_i)^{1/2} \frac{\int B \, dx}{B_{nr}{}^* d} \tag{3}$$

where $B_{nr}{}^*$ is the quantity in (1) without the relativistic correction. To minimize the deflection, it is desirable to confine the magnetic field to the accelerating gap. In this case, the integral in (3) is proportional to the total number of magnetic field lines in the accelerating gap. This is a constant in the self insulating systems described below, even if the field penetrates somewhat into the cathode extraction holes, so that to first order the deflection angle is constant on a microscopic scale.

Since the self magnetic fields of current flowing into the diode are always present, it is of interest to consider under what circumstances these fields can be used in a practical magnetic insulation geometry. This type of insulation has been chiefly associated with the pinched electron beam diode. Pinched electron beams have been studied for some time and the observation of intense ion fluxes has followed from the theoretical discovery that such fluxes were necessary to explain the electron behavior (POUKEY, 1975; GOLDSTEIN *et al.*, 1975). Rather than immediately consider this geometry, the cylindrical diode of Fig. 1(b) will first be treated. In order for the entire diode to be insulated, the current at $z = L$ must be high enough to make $B_\theta(L) > B^*$. For the fast time scales involved, it is not practical to recover the energy associated with this current, so the simplest expedient is to provide the extra current by locating a resistor at the end of the cylinder. This can be of the form of an electron diode with small enough spacing so that the magnetic fields do not impede the flow of electrons. The total current entering the system, $I_z(0)$, is the sum of this electron current, I_e, plus the total ion current, $I_i(0)$. The ion current, $I_i(z)$, decreases with z, making B_θ a minimum at $z = L$. The drifting electron flux on the cathode thus increases with z (electrons can be added along the cylinder to provide an equilibrium), but as long as $B_\theta(L)$ is well above B^* the electron sheath should be confined near the cathode and the ion current density should be close to the Child–Langmuir limit. If it is assumed that $B_\theta(l) = \gamma B^*$ and that the ion current density is determined by simple space charge limited flow, then the ratio of ion current to the necessary electron current is

$$\frac{I_i(0)}{I_e}=\frac{4}{9\gamma}\left[\frac{eV_0}{m_ec^2}\right]\left[\frac{m_e}{m_i}\right]^{1/2}\left[\frac{L}{d}\right]\left[1+\frac{eV_0}{2m_ec^2}\right]^{-1/2} \tag{4}$$

This is close to the result obtained by BLAUGRUND *et al.* (1975) for the pinched electron beam diode. A comparison with (2) shows that typical voltages ($V_0 > m_ec^2$) this current is larger than the maximum electron drift current and thus this is a valid equilibrium situation, where drifting electrons reaching the end of the cylinder are transported across the electron diode. Note that the figure of merit ($I_i(0)/I_e$) improves with increasing voltage, V_0 and aspect ratio, L/d.

In comparing this self insulating geometry to a cylindrical focusing geometry using an applied B_z, there are a number of advantages and drawbacks. The self insulated diode is easier to construct and the magnetic field is automatically constrained to the accelerating gap. This means not only that there is less volume to fill with field, but also that the beam propagation is easier, both from the point of view of providing electron neutralization and minimizing the magnetic deflections of ions. Similar results can be obtained using an applied field with a field exclusion cathode (HUMPHRIES *et al.*, 1976) but at the expense of greater construction difficulties. Since the fields in a self insulated system are applied for only a fraction of a microsecond, the problem of mechanical strength of the electrodes is lessened. An energy utilization figure of merit (beam energy/energy to produce the magnetic field) can be obtained from (4) by multiplying the numerator and denominator by $V\Delta t$, where Δt is the pulse length. The self insulated diode has a relatively poor utilization since the magnetic field circuit consists of an inductor in series with a resistor (the electron diode). The requirement that the L/R time of the diode be short compared to the pulse length implies that most of the energy in the magnetic field circuit is lost to the resistor. With applied fields, on the other hand, the resistive element can easily be made small. Assuming no recovery of the magnetic field energy, the energy utilization factor for this type of system can be written as

$$\frac{I_i(0)V_0\Delta t}{(B^2/8\pi)2\pi r_dLd\beta}\simeq\left[\frac{4}{9\beta\gamma^2}\right]\left[\frac{eV_0}{m_ec^2}\right]\left[2eV_0\quad\right.\left[\frac{\Delta t}{d}\right]\left[1+\frac{eV_0}{2m_ec^2}\right] \tag{5}$$

where r_d is the diode radius and it is assumed that the magnetic field fills a volume β times that of the diode. As an example, for $V_0 = 2$ mV, $\Delta t = 100$ ns, $d = 1$ cm, $\gamma = 2$ and $\beta = 10$, the magnetic circuit energy for an applied field diode is only 4% of the ion beam energy, while for a large aspect ratio self field diode ($L/d = 43$) it is 40%. The applied field also has the advantage that it may provide protection of the diode against postpulse damage. On the other hand, it may be possible to operate self insulated diodes without the extensive damage characteristic of electron pinches. This could be brought about partly by using a lower energy density electron diode as the resistor and partly by providing adequate initial anode plasma so that the system proceeds to an insulated equilibrium directly (EICHENBERGER *et al.*) rather than proceeding through the non-equilibrium stages described by GOLDSTEIN *et al.* (1975).

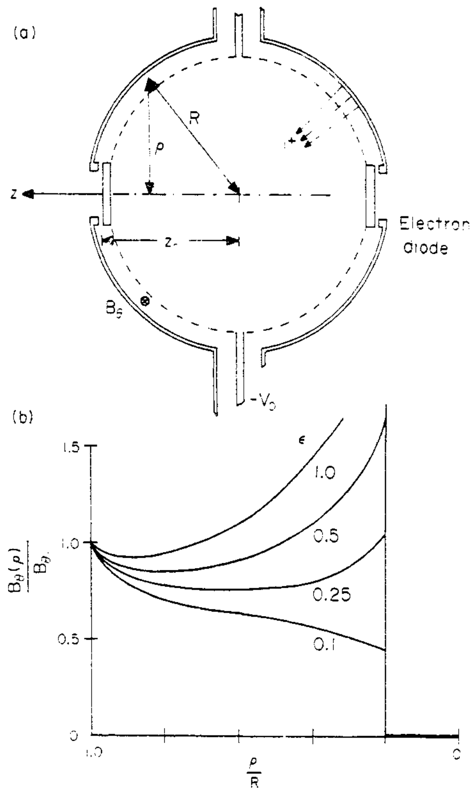

FIG. 2.—(a) Spherical self magnetically insulated ion focusing diode, valid when $\alpha \to 0$. (b) Variation of B_θ as a function of ρ in the spherical diode for $j_i = \text{constant}/\rho$ and $\varepsilon = I_e/I_i(0)$, $z_0/R = 0.98$.

In order to achieve a focused ion beam with a self insulating magnetic diode, it is necessary to shape it roughly as a sphere with radial current feeds at the equator as in Fig. 2(a). A number of ideas have been described by GOLDSTEIN (private communication) and POUKEY *et al.* (1975) for the pinched electron beam diode. In this geometry, a problem arises as to how to treat the drifting electrons. This is, in general, a complex process which combines the local drift speed of the electrons determined by the self consistent electric and magnetic fields with the requirement that $E = 0$ at the cathode and anode. GOLDSTEIN *et al.* (1975) have proposed taking the electron space charge density as proportional to ρ^{-1}. This would come about if the electron cloud was composed of upstream electrons whose drift speed remained relatively constant. This is probably a good assumption for regions outside the pinch since emitted electrons cannot return to the cathode in equilibrium and both B_θ and E tend to increase towards the center ($\rho = 0$). The assumption is in rough agreement with computer simulations (POUKEY, 1975) and experiments (BLAUGRUND *et al.*, 1975; MILLER *et al.*, 1975; EICHENBERGER *et al.*, to be published). In order that $E = 0$ on both electrodes, the ion space charge must increase with the electron space charge in the accelerating gap. This has the effect of causing the ion current density to increase as ρ^{-1}.

Given an assumed variation of ion current density, the variation of B_θ in the accelerating gap of a hemispherical diode can be calculated. The electron current, taken as $I_e = \varepsilon I_i(z=0)$, is constant with z while the ion current varies because of ions crossing the gap. In the case of j_i = constant, the axial current in one hemisphere is given by $I_z(z) = I_i(0)(1+\varepsilon-(z/z_0))$, while if j_i = constant/ρ it becomes $I_z(z) = I_i(0)\times(1+\varepsilon-\sin^{-1}(z/R)/\sin^{-1}(z_0/R))$. For the second case, B_θ expressed as a function of ρ is

$$\frac{B_\theta(\rho)}{B_\theta(z=0)} = \frac{R}{\rho}\left[1-\frac{\sin^{-1}(1-(\rho/R)^2)^{1/2}}{(1+\varepsilon)\sin^{-1}(z_0/R)}\right]. \tag{7}$$

This is plotted in Fig. 2(b) for $z_0/R = 0.98$ and various values of ε. In general, depending on ε, the minimum of B_θ may no longer occur at the electron diode, and this must be taken into account in computing the system figure of merit. For the most part, the figure of merit will be higher when ρ decreases towards the electron diode.

As discussed above, when the ion magnetic deflection in the accelerating gap becomes significant, a sphere is not the ideal shape for focusing applications. The optimum shape can easily be derived if it is assumed that the propagating beam is space charge neutralized so that after leaving the accelerating gap the ions proceed on ballistic paths in a field free region. Using the target centered polar coordinate system (R, ϕ) shown in Fig. 3(a), ideally all ions should approach the target on a radial line. In order to compensate for the magnetic deflection, the diode surfaces should be canted at an angle $\alpha(R, \phi)$ with respect to the appropriate radius. Thus, assuming $d \ll R$, the optimum diode surface is defined by the

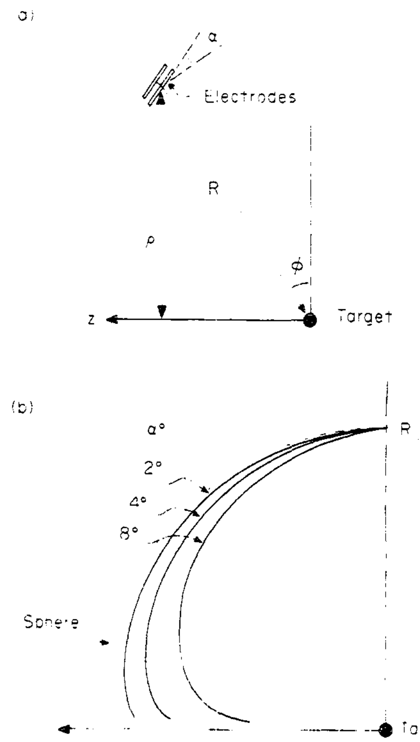

FIG. 3.—(a) Geometry for calculating the optimum electrode shape to produce an ion focus in the presence of a varying magnetic deflection α. (b) Modified surfaces for the self insulated diode of Fig. 2(a) when $\alpha = 0$.

equation

$$\frac{dR}{R\,d\phi} = \alpha(R, \phi). \tag{8}$$

As an example, consider the problem of producing a cylindrical focus in a diode insulated by uniform applied B_z so that the deflection angle is everywhere constant if d is constant. The equation of the surface is $r/r_0 = \exp(\alpha\,\phi)$. For insulation using B_θ fields, the shape to produce a spherical focus is the surface of revolution around the z axis of the curve defined by (8). The equation can be solved numerically given a law to determine j_i and hence $I_z(z)$ and B_θ. An analytic solution can be obtained for the case of electron current only, or I_z = constant. Defining R_0 and α_0 at $z = 0$, the surface equation becomes $dR/R\,d\phi = -\alpha_0 R_0/R \cos\phi$. The solution to this equation, $R = R_0 - \alpha_0 R_0 \ln\tan(\pi/4 + \phi/2)$, is plotted in Fig. 3(b) for a number of values of α_0. At small values of ρ, the magnetic deflection angle becomes large and there may also be significant electrostatic deflection from the distortion of equipotential surfaces due to the buildup of electron space charge (POUKEY *et al.*, 1975). A practical minimum ρ is determined by the maximum deflection angle, α_{max}, that can be accommodated in an extractor design with good microscopic divergence properties. For the case in question, the minimum ρ can be expressed as

$$\rho_{min} = R_0\alpha_0/\alpha_{max}. \tag{9}$$

For example, with $\alpha_0 = 2°$, $\alpha_{max} = 20°$ and $R_0 = 2$ m, ρ_{min} is found to be 20 cm, much larger than a typical electron beam pinch.

In conclusion, it has been shown that self magnetically insulated pulsed ion diodes have a number of experimental attractions compared to those using applied magnetic fields. In comparing self magnetically insulated systems for focusing applications, the pinched electron beam diode may not be the most favorable configuration because (1) the pinch causes electrode damage, (2) the pinch raises the total diode inductance, (3) the electron flow cannot be as easily controlled as in a simple Child–Langmuir limited electron diode and (4) ions from the vicinity of the pinch probably cannot be focused at the target.

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